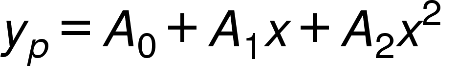
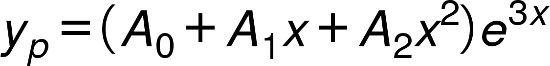
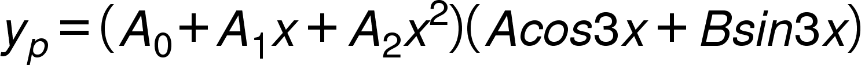
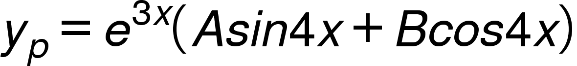
Method of undetermine Co-efficient

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3. {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>3</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup><mml:mi>s</mml:mi><mml:mi>i</mml:mi><mml:mi>n</mml:mi><mml:mn>4</mml:mn><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} 

Note: If any point is is already in y

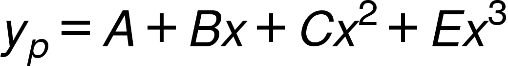
Then multiply by x or

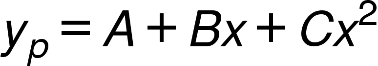
M = 2,2

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Case (1) – Q(x) = polynomial function

x

 {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>p</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:mi>A</mml:mi><mml:mo>+</mml:mo><mml:mi>B</mml:mi><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}



Case(2) - Q(x) = expolynomial function

= {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mi>a</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

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Case(3) - {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>Q</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>)</mml:mo><mml:mo>=</mml:mo><mml:mi>s</mml:mi><mml:mi>i</mml:mi><mml:mi>n</mml:mi><mml:mo>/</mml:mo><mml:mi>c</mml:mi><mml:mi>o</mml:mi><mml:mi>s</mml:mi><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} function

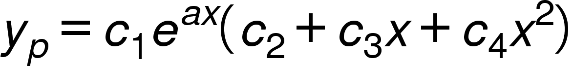
i.e {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>Q</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>)</mml:mo><mml:mo>=</mml:mo><mml:mi>s</mml:mi><mml:mi>i</mml:mi><mml:mi>n</mml:mi><mml:mi>a</mml:mi><mml:mi>x</mml:mi><mml:mo>/</mml:mo><mml:mi>c</mml:mi><mml:mi>o</mml:mi><mml:mi>s</mml:mi><mml:mi>a</mml:mi><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

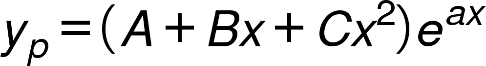
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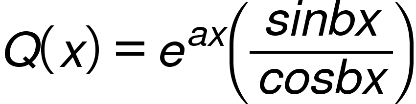
Case(4) - {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>Q</mml:mi><mml:mfenced separators=\"|\"><mml:mi>x</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mi>a</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:msup><mml:mo>(</mml:mo><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mn>1</mml:mn><mml:mo>)</mml:mo></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Here, e= Exponential x= polynomias

Function function





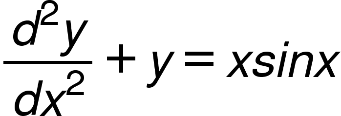
Case(5) - 

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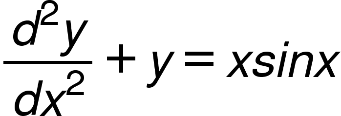
Case(6) – Q(x) = polynomial function

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Ross-21: 

Solution: Given that

 ………………….(1)

Let {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mi>m</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} be the solution of {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>y</mml:mi><mml:mo>&quot;</mml:mo><mml:mo>+</mml:mo><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Then the A.E is {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>m</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mn>1</mml:mn><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>m</mml:mi><mml:mo>=</mml:mo><mml:mo>&#xB1;</mml:mo><mml:mi>i</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

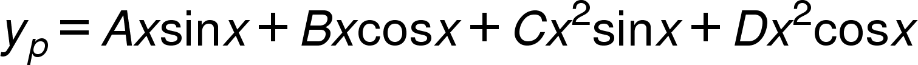
The complementary function

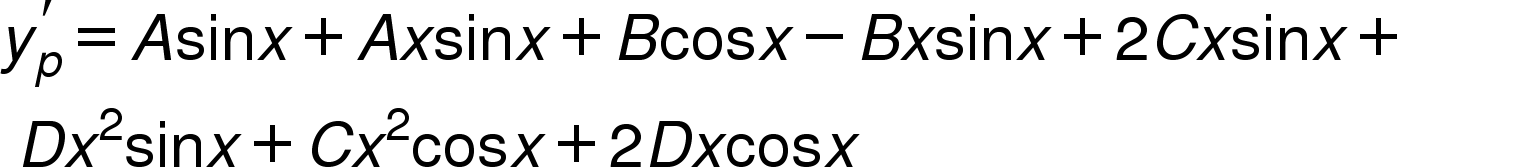
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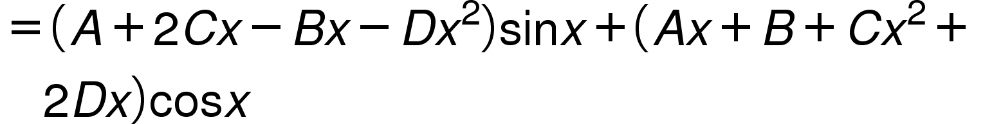
By the method of undetermined Co-efficient

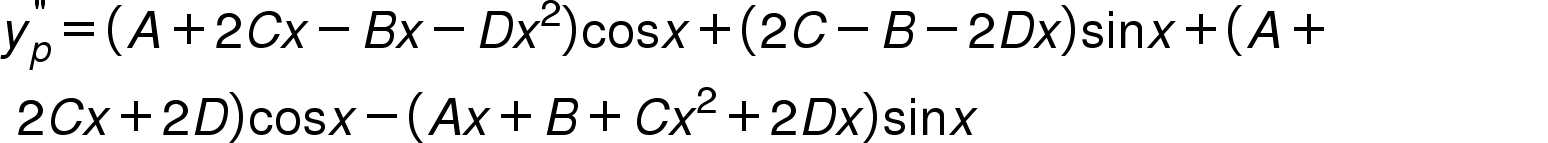
We get,

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mo>-</mml:mo><mml:mn>2</mml:mn><mml:mi>B</mml:mi><mml:mi>s</mml:mi><mml:mi>i</mml:mi><mml:mi>n</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>F</mml:mi><mml:mi>cos</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>=</mml:mo><mml:mi>x</mml:mi><mml:mi>sin</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}



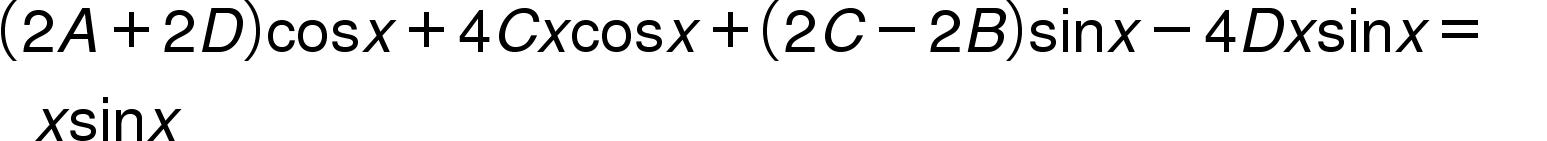






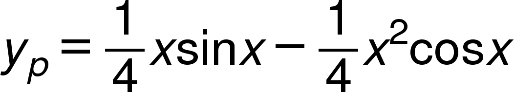
Putting this value of(1) , we get

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mfenced separators=\"|\"><mml:mrow><mml:mi>A</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>C</mml:mi><mml:mi>x</mml:mi><mml:mo>-</mml:mo><mml:mi>B</mml:mi><mml:mi>x</mml:mi><mml:mo>-</mml:mo><mml:mi>D</mml:mi><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup></mml:mrow></mml:mfenced><mml:mi>cos</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mfenced separators=\"|\"><mml:mrow><mml:mn>2</mml:mn><mml:mi>C</mml:mi><mml:mo>-</mml:mo><mml:mi>B</mml:mi><mml:mo>-</mml:mo><mml:mn>2</mml:mn><mml:mi>D</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:mfenced><mml:mi>sin</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mspace linebreak=\"newline\"/><mml:mfenced separators=\"|\"><mml:mrow><mml:mi>A</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>C</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>D</mml:mi></mml:mrow></mml:mfenced><mml:mi>cos</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>-</mml:mo><mml:mfenced separators=\"|\"><mml:mrow><mml:mi>A</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>B</mml:mi><mml:mo>+</mml:mo><mml:mi>C</mml:mi><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>D</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:mfenced><mml:mi>sin</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>A</mml:mi><mml:mi>x</mml:mi><mml:mi>sin</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mspace linebreak=\"newline\"/><mml:mi>B</mml:mi><mml:mi>x</mml:mi><mml:mi>cos</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>C</mml:mi><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mi>sin</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>D</mml:mi><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mi>cos</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>=</mml:mo><mml:mi>x</mml:mi><mml:mi>sin</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mfrac><mml:mrow/><mml:mrow/></mml:mfrac></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}



Comparing on both sides, we get

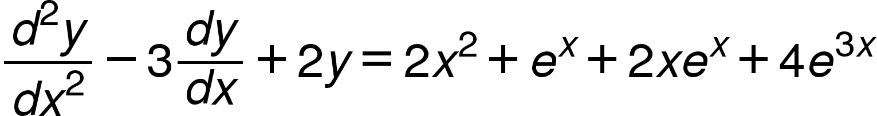
-4D = 1 , D = - 2A+2D , A =

Thus the particular interal 

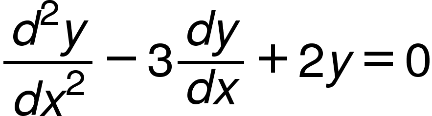
Hence the general solution is

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>+</mml:mo><mml:msub><mml:mi>y</mml:mi><mml:mi>p</mml:mi></mml:msub></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

4.37-



Solution: let {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mi>m</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} be the trial solution of

 Then the A.E is

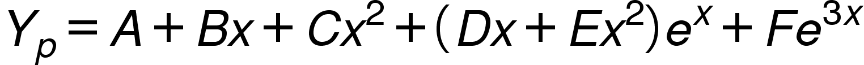
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>m</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>-</mml:mo><mml:mn>3</mml:mn><mml:mi>m</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

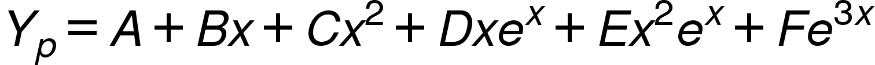
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mo>(</mml:mo><mml:mi>m</mml:mi><mml:mo>-</mml:mo><mml:mn>1</mml:mn><mml:mo>)</mml:mo><mml:mo>(</mml:mo><mml:mi>m</mml:mi><mml:mo>-</mml:mo><mml:mn>2</mml:mn><mml:mo>)</mml:mo><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

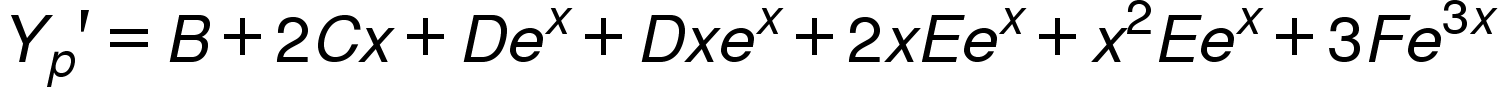
m= 1, 2

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>1</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>2</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>2</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

*By the method of undetermined co efficient we set:*



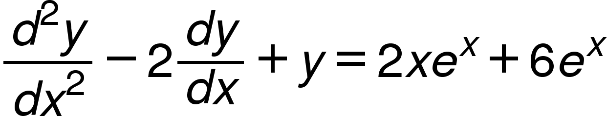
 x

+ {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mrow><mml:mi>E</mml:mi><mml:mi>x</mml:mi></mml:mrow><mml:mn>2</mml:mn></mml:msup><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>Y</mml:mi><mml:mi>p</mml:mi></mml:msub><mml:mo>''</mml:mo><mml:mo>=</mml:mo><mml:mo>.</mml:mo><mml:mo>.</mml:mo><mml:mo>.</mml:mo></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

A, B, C, D, E, F

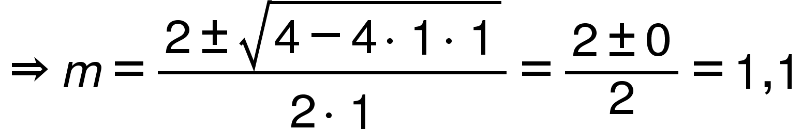
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>+</mml:mo><mml:msub><mml:mi>y</mml:mi><mml:mi>p</mml:mi></mml:msub></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

…….(1)

Let {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mi>m</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} *be the trial solution of*

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>D</mml:mi><mml:msup><mml:mi>y</mml:mi><mml:mrow><mml:mi>'</mml:mi><mml:mi>'</mml:mi></mml:mrow></mml:msup><mml:mo>-</mml:mo><mml:mn>2</mml:mn><mml:mi>y</mml:mi><mml:mi>'</mml:mi><mml:mo>+</mml:mo><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}, *then the A,E*

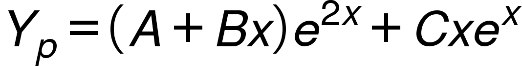
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>m</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>-</mml:mo><mml:mn>2</mml:mn><mml:mi>m</mml:mi><mml:mo>+</mml:mo><mml:mn>1</mml:mn><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

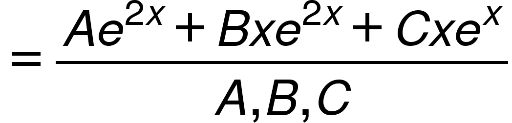


So,

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:mn>4</mml:mn><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>x</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

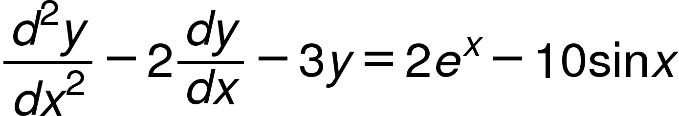
*By the undetermined coefficient, we set:*



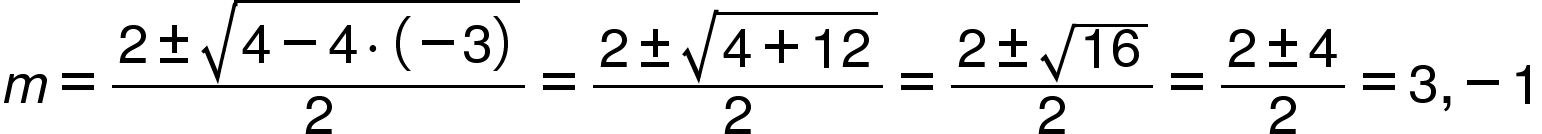


{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>+</mml:mo><mml:msub><mml:mi>y</mml:mi><mml:mi>p</mml:mi></mml:msub></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**4.39**



Let {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mi>m</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} *be the trial solution of*

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>y</mml:mi><mml:mo>''</mml:mo><mml:mo>-</mml:mo><mml:mn>2</mml:mn><mml:mi>y</mml:mi><mml:mi mathvariant=\"normal\">'</mml:mi><mml:mo>-</mml:mo><mml:mn>3</mml:mn><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn><mml:mo>&#x21D2;</mml:mo><mml:mtext>Then&#xA0;the&#xA0;A.E.&#xA0;is</mml:mtext></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>m</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>-</mml:mo><mml:mn>2</mml:mn><mml:mi>m</mml:mi><mml:mo>-</mml:mo><mml:mn>3</mml:mn><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} 

*The complementary function:*

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>1</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>3</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup><mml:mo>+</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>2</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mo>-</mml:mo><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

*By the method of undetermined coefficients, we set:*

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>p</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:mi>B</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>C</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Then:

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>p</mml:mi></mml:msub><mml:mi mathvariant=\"normal\">'</mml:mi><mml:mo>=</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>-</mml:mo><mml:mi>B</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>C</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

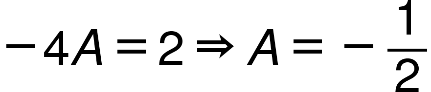
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>p</mml:mi></mml:msub><mml:mo>''</mml:mo><mml:mo>=</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>-</mml:mo><mml:mi>B</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi>x</mml:mi><mml:mo>-</mml:mo><mml:mi>C</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

*Putting these values in (1):*

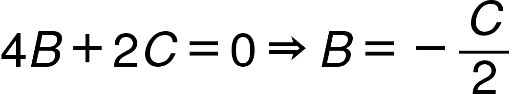
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>-</mml:mo><mml:mi>B</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi>x</mml:mi><mml:mo>-</mml:mo><mml:mi>C</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi><mml:mo>-</mml:mo><mml:mn>2</mml:mn><mml:mi>A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi mathvariant=\"normal\">B</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi mathvariant=\"normal\">x</mml:mi><mml:mo>-</mml:mo><mml:mn>2</mml:mn><mml:mi mathvariant=\"normal\">C</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">x</mml:mi><mml:mo>-</mml:mo><mml:mn>3</mml:mn><mml:mi mathvariant=\"normal\">A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>-</mml:mo><mml:mspace linebreak=\"newline\"/><mml:mn>3</mml:mn><mml:mi mathvariant=\"normal\">B</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">x</mml:mi><mml:mo>-</mml:mo><mml:mn>3</mml:mn><mml:mi mathvariant=\"normal\">C</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi mathvariant=\"normal\">x</mml:mi><mml:mo>=</mml:mo><mml:mn>2</mml:mn><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>-</mml:mo><mml:mn>10</mml:mn><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi mathvariant=\"normal\">x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

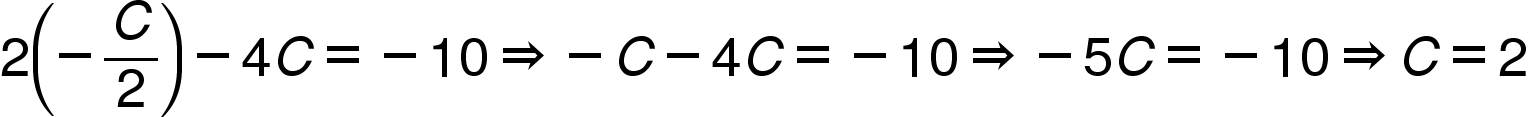
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*Comparing on both sides, we get:*

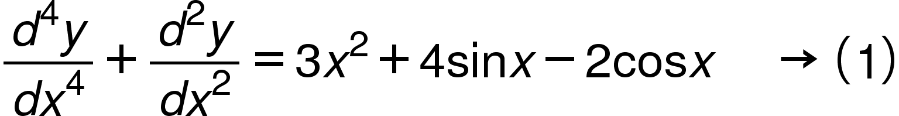
* 
* {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mo>-</mml:mo><mml:mfenced separators=\"|\"><mml:mrow><mml:mn>4</mml:mn><mml:mi>B</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>C</mml:mi></mml:mrow></mml:mfenced><mml:mo>=</mml:mo><mml:mn>0</mml:mn><mml:mo>&#x21D2;</mml:mo><mml:mn>4</mml:mn><mml:mi>B</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>C</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}
* {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mn>2</mml:mn><mml:mi>B</mml:mi><mml:mo>-</mml:mo><mml:mn>4</mml:mn><mml:mi>C</mml:mi><mml:mo>=</mml:mo><mml:mo>-</mml:mo><mml:mn>10</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

*Solving:*

* From 
* Plug into {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mn>2</mml:mn><mml:mi>B</mml:mi><mml:mo>-</mml:mo><mml:mn>4</mml:mn><mml:mi>C</mml:mi><mml:mo>=</mml:mo><mml:mo>-</mml:mo><mml:mn>10</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}:

 and {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>B</mml:mi><mml:mo>=</mml:mo><mml:mo>-</mml:mo><mml:mn>1</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

4.38  
Ross–149

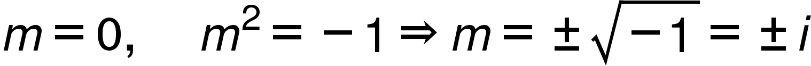


Let {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mi>m</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} *be the solution of*

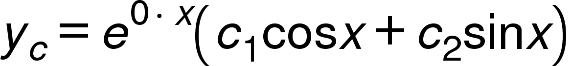
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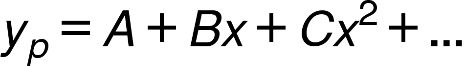
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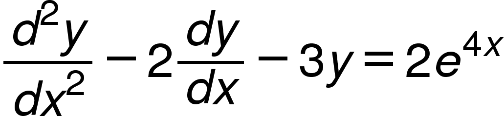
*Then the complementary function:*



*By the method of undetermined coefficients also:*



Ross R–138  
4.29

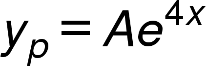


Auxiliary equation:

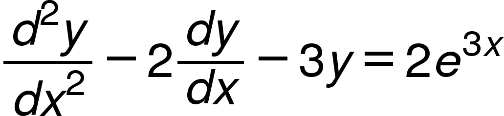
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*By the method of undetermined coefficients, use*



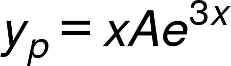
(Below that)

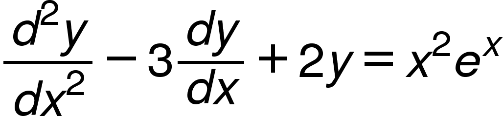


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*Since RHS is* {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>3</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}*, and* {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>3</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} *is in , we multiply by :*

**4.35**

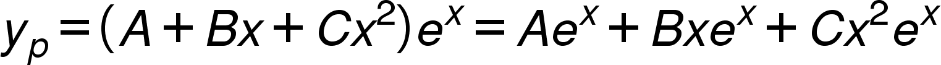
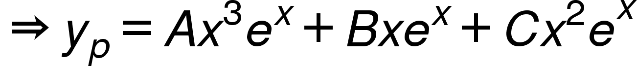


*Auxiliary equation:*

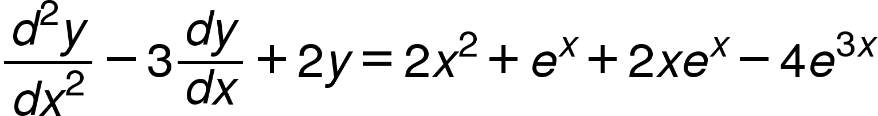
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>m</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>-</mml:mo><mml:mn>3</mml:mn><mml:mi>m</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mo>=</mml:mo><mml:mn>0</mml:mn><mml:mo>&#x21D2;</mml:mo><mml:mi>m</mml:mi><mml:mo>=</mml:mo><mml:mn>1</mml:mn><mml:mo>,</mml:mo><mml:mn>2</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

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*Particular integral:*

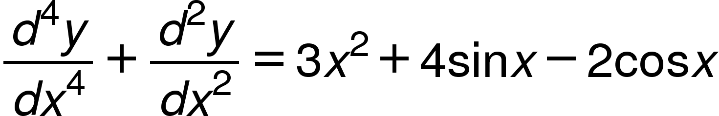
**4.37**



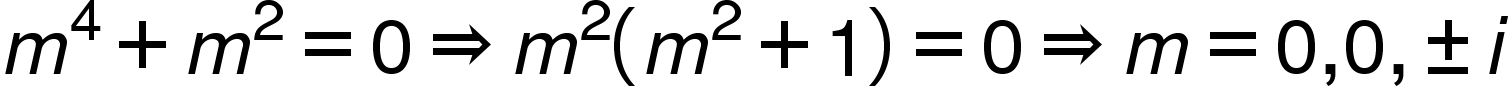
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>1</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>2</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>2</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

### {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>p</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mi>B</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>C</mml:mi><mml:mo>+</mml:mo><mml:mfenced separators=\"|\"><mml:mrow><mml:mi>A</mml:mi><mml:mo>+</mml:mo><mml:mi>B</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:mfenced><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>3</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mo>=</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mi>B</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>C</mml:mi><mml:mo>+</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:mi>B</mml:mi><mml:mi>x</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>3</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:menclose notation=\"box\"><mml:msub><mml:mi>y</mml:mi><mml:mi>p</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mi>B</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>C</mml:mi><mml:mo>+</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mrow><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mi>e</mml:mi></mml:mrow><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:mi>B</mml:mi><mml:mi>x</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>3</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:menclose></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

### *Problem* ***4.38****:*



*Auxiliary Equation:*



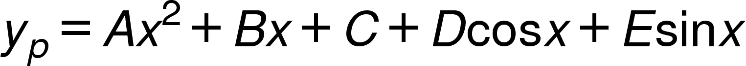
*So,*

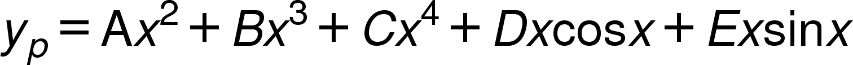
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>1</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>0</mml:mn><mml:mo>.</mml:mo><mml:mi>x</mml:mi></mml:mrow></mml:msup><mml:mo>+</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>2</mml:mn></mml:msub><mml:mi>x</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>0</mml:mn><mml:mo>.</mml:mo><mml:mi>x</mml:mi></mml:mrow></mml:msup><mml:mo>+</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>3</mml:mn></mml:msub><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>4</mml:mn></mml:msub><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

*Simplified:*

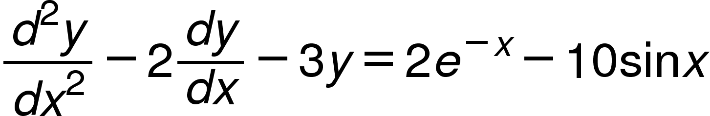
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>1</mml:mn></mml:msub><mml:mo>+</mml:mo><mml:mi>x</mml:mi><mml:msub><mml:mi>c</mml:mi><mml:mn>2</mml:mn></mml:msub><mml:mo>+</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>3</mml:mn></mml:msub><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>4</mml:mn></mml:msub><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

*Particular integral:*





### *Problem* ***4.39****:*



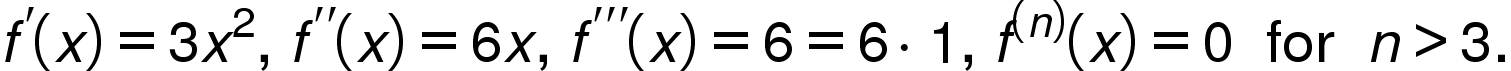
*Auxiliary Equation:*

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>m</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>-</mml:mo><mml:mn>2</mml:mn><mml:mi>m</mml:mi><mml:mo>-</mml:mo><mml:mn>3</mml:mn><mml:mo>=</mml:mo><mml:mn>0</mml:mn><mml:mo>&#x21D2;</mml:mo><mml:mi>m</mml:mi><mml:mo>=</mml:mo><mml:mo>-</mml:mo><mml:mn>1</mml:mn><mml:mo>,</mml:mo><mml:mn>3</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>1</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mo>-</mml:mo><mml:mi>x</mml:mi></mml:mrow></mml:msup><mml:mo>+</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>2</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>3</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>p</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:mi>A</mml:mi><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>+</mml:mo><mml:mi>B</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>C</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Example 4.31**  
The function defined for all real by is a UC function. Computing derivatives of , we find



The linearly independent UC functions of which the successive derivatives of are either constant multiples or linear combinations are those given by

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>,</mml:mo><mml:mi mathvariant=\"normal\">&#xA0;</mml:mi><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi mathvariant=\"normal\">&#xA0;</mml:mi><mml:mn>1</mml:mn><mml:mo>.</mml:mo></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Thus the of is the set {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>S</mml:mi><mml:mo>=</mml:mo><mml:mfenced open=\"{\" close=\"}\" separators=\"|\"><mml:mrow><mml:msup><mml:mi>x</mml:mi><mml:mn>3</mml:mn></mml:msup><mml:mo>,</mml:mo><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>,</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mn>1</mml:mn></mml:mrow></mml:mfenced></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}.

**Example 4.32**  
The function defined for all real by {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>f</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>)</mml:mo><mml:mo>=</mml:mo><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mn>2</mml:mn><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} is a UC function. Computing derivatives of , we find

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>f</mml:mi><mml:mi>'</mml:mi></mml:msup><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>)</mml:mo><mml:mo>=</mml:mo><mml:mn>2</mml:mn><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mn>2</mml:mn><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi mathvariant=\"normal\">&#xA0;</mml:mi><mml:msup><mml:mi>f</mml:mi><mml:mrow><mml:mi>'</mml:mi><mml:mi>'</mml:mi></mml:mrow></mml:msup><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>)</mml:mo><mml:mo>=</mml:mo><mml:mo>-</mml:mo><mml:mn>4</mml:mn><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mn>2</mml:mn><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi mathvariant=\"normal\">&#xA0;</mml:mi><mml:mo>&#x2026;</mml:mo><mml:mo>.</mml:mo></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

The only linearly independent UC function of which the successive derivatives of are constant multiples or linear combinations is that given by . Thus the UC set of is the set {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>S</mml:mi><mml:mo>=</mml:mo><mml:mo>{</mml:mo><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mn>2</mml:mn><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mn>2</mml:mn><mml:mi>x</mml:mi><mml:mo>}</mml:mo></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}.

These and similar examples of the four basic types of UC functions lead to the results listed as numbers 1,2, and 3 of Table 4.1.

**Example 4.33**  
The function defined for all real by {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>f</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>)</mml:mo><mml:mo>=</mml:mo><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} is the product of the two UC functions defined by and . Hence is itself a UC function. Computing derivatives of , we find

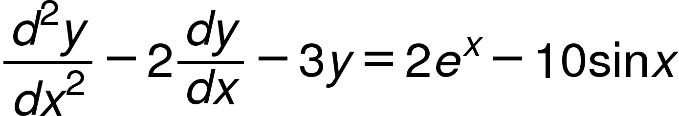
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No "new" types of functions will occur from further differentiation. Each derivative of is a linear combination of certain of the six UC functions given by {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"},

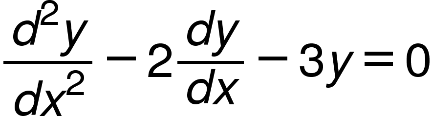
**Example 4.34**The function defined for all real by {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>f</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>)</mml:mo><mml:mo>=</mml:mo><mml:msup><mml:mi>x</mml:mi><mml:mn>3</mml:mn></mml:msup><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mn>2</mml:mn><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} is the product of the two UC functions defined by and . Using the result stated in the preceding paragraph, the UC set of this product is the set of all products obtained by multiplying the various members of the UC set of by the various members of the UC set of . Using the definition of UC set or the appropriate numbers of Table 4.1, we find that the set of is

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mfenced open=\"{\" close=\"}\" separators=\"|\"><mml:mrow><mml:msup><mml:mi>x</mml:mi><mml:mn>3</mml:mn></mml:msup><mml:mo>,</mml:mo><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>,</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mn>1</mml:mn></mml:mrow></mml:mfenced></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Example 4.36**



The corresponding homogeneous equation is

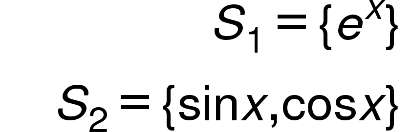


and the complementary function is

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>y</mml:mi><mml:mi>c</mml:mi></mml:msub><mml:mo>=</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>1</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mn>3</mml:mn><mml:mi>x</mml:mi></mml:mrow></mml:msup><mml:mo>+</mml:mo><mml:msub><mml:mi>c</mml:mi><mml:mn>2</mml:mn></mml:msub><mml:msup><mml:mi>e</mml:mi><mml:mrow><mml:mo>-</mml:mo><mml:mi>x</mml:mi></mml:mrow></mml:msup></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

The nonhomogenous term is the linear combination {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mn>2</mml:mn><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>-</mml:mo><mml:mn>10</mml:mn><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} of the two UC functions given by and .

1. Form the UC set for each of these two functions. We find



1. Note that neither of these sets is identical with nor included in the other; hence both are retained.
2. Furthermore, by examining the complementary function, we see that none of the functions in either of these sets is a solution of the corresponding homogeneous equation. Hence neither set needs to be revised.
3. Thus the original sets and remain intact in this problem, and we form the linear combination

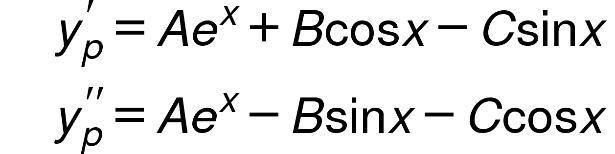
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of the three elements {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msup><mml:mi>e</mml:mi><mml:mi>x</mml:mi></mml:msup><mml:mo>,</mml:mo><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mo>&#x2061;</mml:mo><mml:mi>x</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} of and , with the undetermined coefficients .

5. We determine these unknown coefficients by substituting the linear combination formed in Step 4 into the differential equation and demanding that it satisfy the differential equation identically. That is, we take

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as a particular solution. Then



Actually substituting, we find

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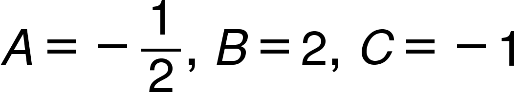
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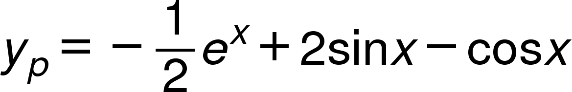
Since the solution is to satisfy the differential equation identically for all on some real interval, this relation must be an identity for all such and hence the coefficients of like terms on both sides must be respectively equal. Equating coefficients of these like terms, we obtain the equations

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From these equations, we find that



and hence we obtain the particular integral



Thus the general solution of the differential equation under consideration is

